Section 1.2 Limits part 2 (Minimum homework: 1 – 29 odds, 33, 35, 37)

#1-20: Find the following limits using Algebra.

1) $\lim_{x\to 5}(2x+6)$ $= 2(5)+6$
$= 16$

3) $\lim_{x\to 3}(x^2+5x-4)$ $= (3)^2 + 5(3) - 4$
$= 9 + 15 - 4$
$= 20$

5) $\lim_{x\to 4}\sqrt{x+5}$ $= \sqrt{4+5} = \sqrt{9} = 3$

7) $\lim_{x\to 2}\frac{3x+6}{x-5} = \frac{3(2)+6}{2-5} = \frac{6+6}{-3} = \frac{12}{-3} = -4$

9) $\lim_{x\to -2}\frac{x^2+5x+6}{x^2+3x+2} = \frac{(-2)^2+5(-2)+6}{(-2)^2+3(-2)+2} = \frac{4-10+6}{4-6+2} = \frac{0}{0}$

Algebra needed

$$= \lim_{x\to -2} \frac{(x+2)(x+3)}{(x+2)(x+1)}$$

$$= \lim_{x\to -2} \frac{x+3}{x+1} = \frac{-2+3}{-2+1} = \frac{1}{-1} = -1$$

11) $\lim_{x\to 3} \frac{x^2-9}{x^2-4x+3}$ $= \frac{(3)^2-9}{(3)^2-4(3)+3} = \frac{9-9}{9-12+3} = \frac{0}{0}$

Algebra needed

$$= \lim_{x\to 3} \frac{(x+3)(x-3)}{(x-1)(x-3)}$$

$$= \lim_{x\to 3} \frac{x+3}{x-1} = \frac{3+3}{3-1} = \frac{6}{2} = 3$$

13) $\lim_{x\to 2} \frac{2x^2-3x-2}{3x^2-2x-8} = \frac{2(2)^2-3(2)-2}{3(2)^2-2(2)-8} = \frac{0}{0}$

Algebra Needed

$2x^2-3x-2$

$= (2x^2-4x)+(1x-2)$

$= 2x(x-2)+1(x-2)$

$= (x-2)(2x+1)$

$3x^2-2x-8$

$= (3x^2-6x)+(4x-8)$

$= 3x(x-2)+4(x-2)$

$= (x-2)(3x+4)$

$$\lim_{x\to 2} \frac{(x-2)(2x+1)}{(x-2)(3x+4)}$$

$$= \lim_{x\to 2} \frac{2x+1}{3x+4} = \frac{2(2)+1}{3(2)+4} = \frac{5}{10} = \frac{1}{2}$$

15) $\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} = \frac{\sqrt{16}-4}{16-16} = \frac{0}{0}$

Algebra Needed

$$\lim_{x\to 16} \frac{(\sqrt{x}-4)(\sqrt{x}+4)}{(x-16)(\sqrt{x}+4)} \rightarrow \sqrt{x}\sqrt{x} + 4\sqrt{x} - 4\sqrt{x} - 16$$

$$= \lim_{x\to 16} \frac{x-16}{(x-16)(\sqrt{x}+4)}$$

$$= \lim_{x\to 16} \frac{1}{\sqrt{x}+4} = \frac{1}{\sqrt{16}+4} = \frac{1}{4+4}$$

$$= 1/8$$

17) $\lim_{x\to 36} \frac{\sqrt{x}-6}{x-36} = \frac{\sqrt{36}-6}{36-36} = \frac{0}{0}$, Algebra needed

$$\lim_{x\to 36} \frac{(\sqrt{x}-6)(\sqrt{x}+6)}{(x-36)(\sqrt{x}+6)}$$

$\sqrt{x}\sqrt{x} = x$, $+6\sqrt{x}-6\sqrt{x}-36 = x-36$

$$= \lim_{x\to 36} \frac{x-36}{(x-36)(\sqrt{x}+6)}$$

$$= \lim_{x\to 36} \frac{1}{\sqrt{x}+6} = \frac{1}{\sqrt{36}+6} = \frac{1}{6+6}$$

$$= \frac{1}{12}$$

19) $\lim_{x\to 121} \frac{\sqrt{x}-11}{x-121} = \frac{\sqrt{121}-11}{121-121} = \frac{0}{0}$ Algebra Needed

$$= \lim_{x\to 121} \frac{(\sqrt{x}-11)(\sqrt{x}+11)}{(x-121)(\sqrt{x}+11)}$$

$\sqrt{x}\sqrt{x}$
$x+11\sqrt{x}-11\sqrt{x}-121$
$x-121$

$$= \lim_{x\to 121} \frac{x-121}{(x-121)(\sqrt{x}+11)}$$

$$= \lim_{x\to 121} \frac{1}{\sqrt{x}+11} = \frac{1}{\sqrt{121}+11} = \frac{1}{11+11}$$

$$= 1/22$$

#21-32: Find the following limits using Algebra.

21) $\lim_{x\to\infty} \frac{3x+6}{2x-4}$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{x}}\left(\frac{3x+6}{2x-4}\right)$$

$$= \lim_{x\to\infty} \frac{3x\cdot\frac{1}{x} + 6\cdot\frac{1}{x}}{2x\cdot\frac{1}{x} - 4\cdot\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{3 + \frac{6}{x}}{2 - \frac{4}{x}} = \frac{3+\frac{6}{\infty}}{2-\frac{4}{\infty}}$$

$$= \frac{3+0}{2-0}$$

$$= 3/2$$

23) $\lim_{x\to\infty}\left(\frac{4x^2-3x+6}{5x^2+2x-4}\right)^{\frac{1/x^2}{1/x^2}}$

$$= \lim_{x\to\infty} \frac{4x^2\cdot\frac{1}{x^2} - 3x\cdot\frac{1}{x^2} + 6\cdot\frac{1}{x^2}}{5x^2\cdot\frac{1}{x^2} + 2x\cdot\frac{1}{x^2} - 4\cdot\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{4 - 3/x + 6/x^2}{5 + 2/x - 4/x^2}$$

$$= \frac{4 - 3/\infty + 6/\infty^2}{5 + 2/\infty - 4/\infty^2}$$

$$= \frac{4 - 0 + 0}{5 + 0 - 0}$$

$$= 4/5$$

25) $\lim_{x\to\infty}\left(\frac{3x+6}{2x^2-4}\right)^{\frac{1/x^2}{1/x^2}}$

$$= \lim_{x\to\infty} \frac{3x\cdot\frac{1}{x^2}+6\cdot\frac{1}{x^2}}{2x^2\cdot\frac{1}{x^2}-4\cdot 1/x^2}$$

$$= \lim_{x\to\infty} \frac{3/x+6/x^2}{2-4/x^2}$$

$$= \frac{3/\infty+6/\infty^2}{2-4/\infty^2}$$

$$= \frac{0+0}{2-0}$$

$$= 0/2$$

$$= 0$$

27) $\lim_{x\to\infty}\left(\frac{4x^2-3x+6}{5x^3+2x-4}\right)^{\frac{1/x^3}{1/x^3}}$

$$= \lim_{x\to\infty} \frac{4x^2\cdot\frac{1}{x^3} - 3x\cdot\frac{1}{x^3} + 6\cdot 1/x^3}{5x^3\cdot\frac{1}{x^3} + 2x\cdot\frac{1}{x^3} - 4\cdot 1/x^3}$$

$$= \lim_{x\to\infty} \frac{\frac{4}{x} - \frac{3}{x^2} + \frac{6}{x^3}}{5 + \frac{2}{x^2} - \frac{4}{x^3}}$$

$$= \frac{\frac{4}{\infty} - \frac{3}{\infty^2} + \frac{6}{\infty^3}}{5 + \frac{2}{\infty^2} - \frac{4}{\infty^3}}$$

$$= \frac{0 - 0 + 0}{5 + 0 - 0} = \frac{0}{5} = 0$$

29) $\lim_{x\to\infty}\left(\frac{3x^2+6}{2x-4}\right)^{1/x^2}$

$$= \lim_{x\to\infty} \frac{3x^2\cdot\frac{1}{x^2}+6\cdot\frac{1}{x^2}}{2x\cdot\frac{1}{x^2}-4\cdot\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{3+6/x^2}{2/x-4/x^2}$$

$$= \frac{3+6/\infty^2}{2/\infty-4/\infty^2}$$

$$= \frac{3+0}{0-0} = \frac{3}{0} = +\infty$$

Sketch graph to confirm sign

31) $\lim_{x\to\infty} \frac{4x^3-3x+6}{5x^2+2x-4}$

$$= \lim_{x\to\infty} \frac{4x^3\cdot\frac{1}{x^3} - 3x\cdot\frac{1}{x^3} + 6\cdot\frac{1}{x^3}}{5x^2\cdot\frac{1}{x^3} + 2x\cdot\frac{1}{x^3} - 4\cdot\frac{1}{x^3}}$$

$$= \lim_{x\to\infty} \frac{4 - 3/x^2 + 6/x^3}{5/x + 2/x^2 - 4/x^3}$$

$$= \frac{4 - 3/\infty^2 + 6/\infty^3}{5/\infty + 2/\infty^2 - 4/\infty^3}$$

$$= \frac{4-0+0}{0+0-0}$$

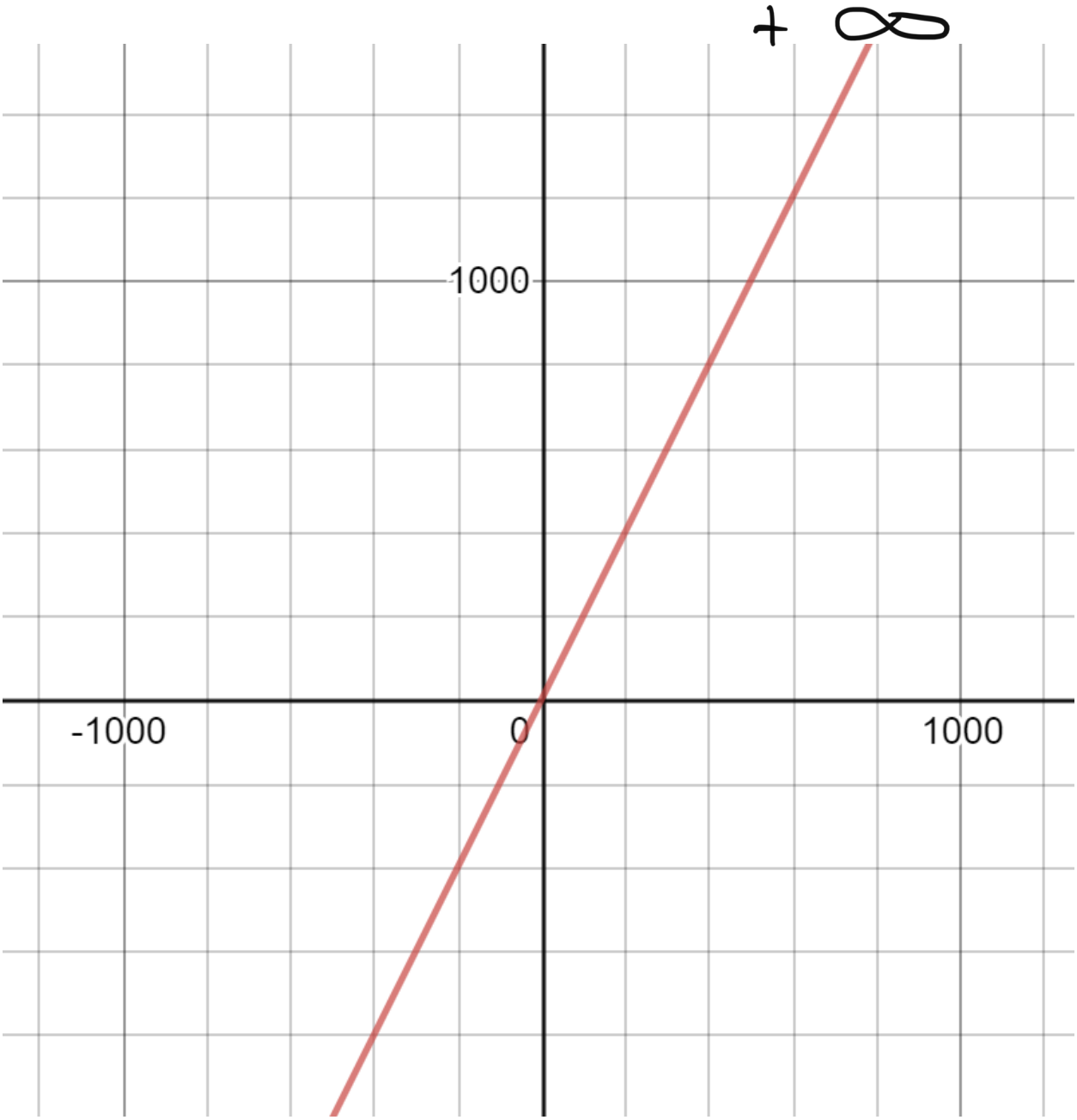
$$= 4/0$$

$$= \infty$$

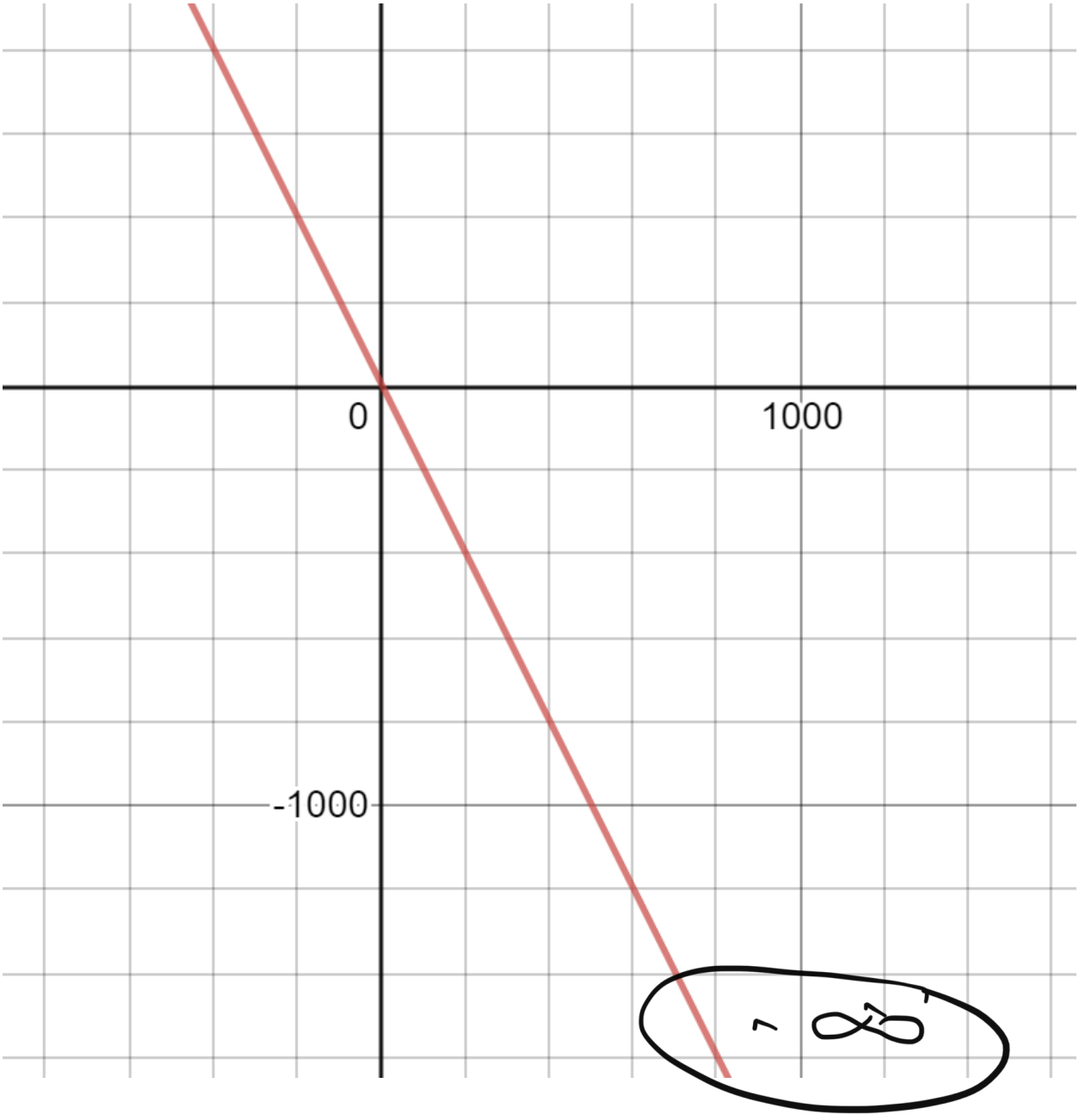
Sketch graph To Determine Sign

#33-40: Find the following limits using Algebra.

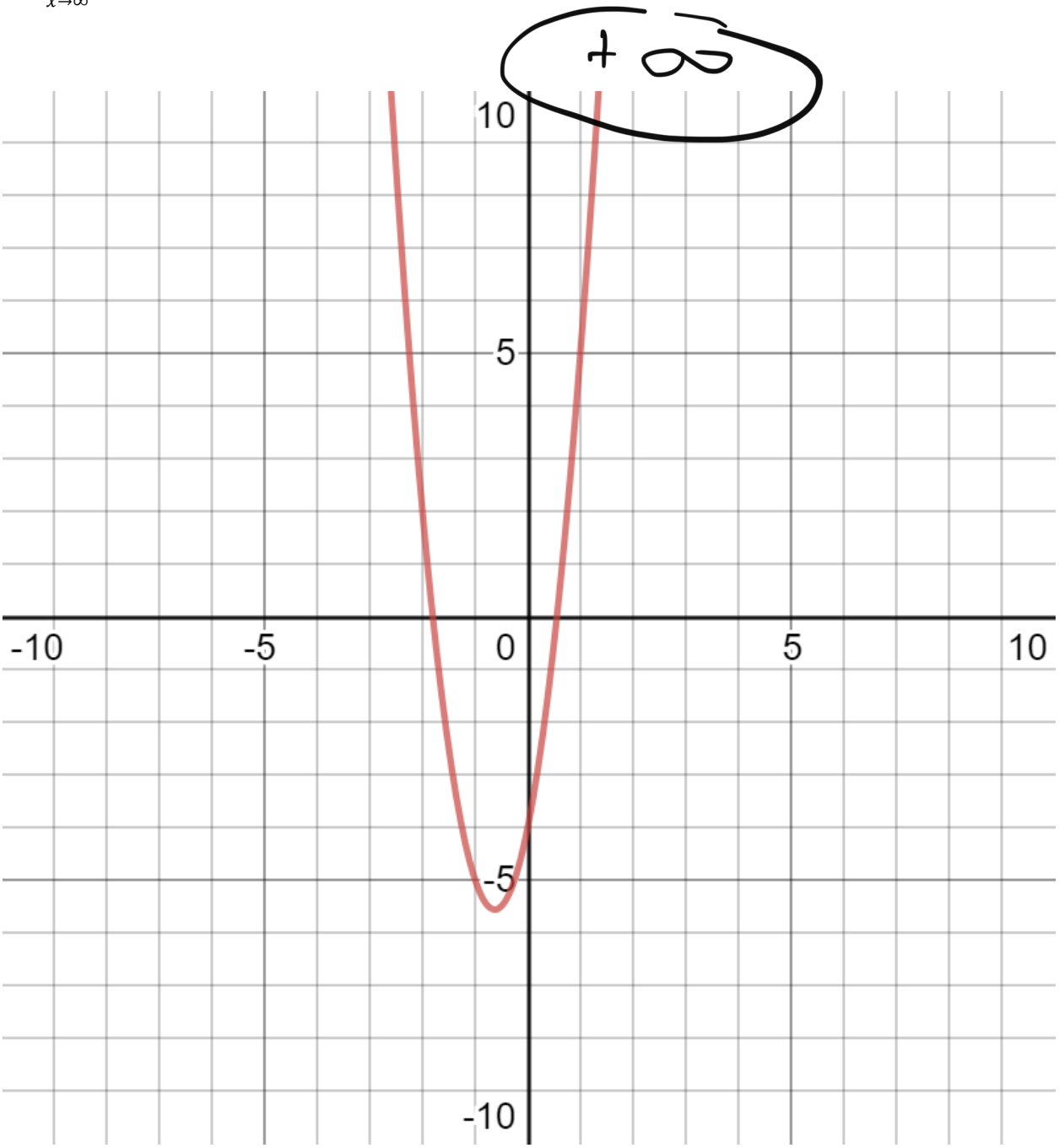
33) $\lim_{x\to\infty}(2x+6)$ $= +\infty$



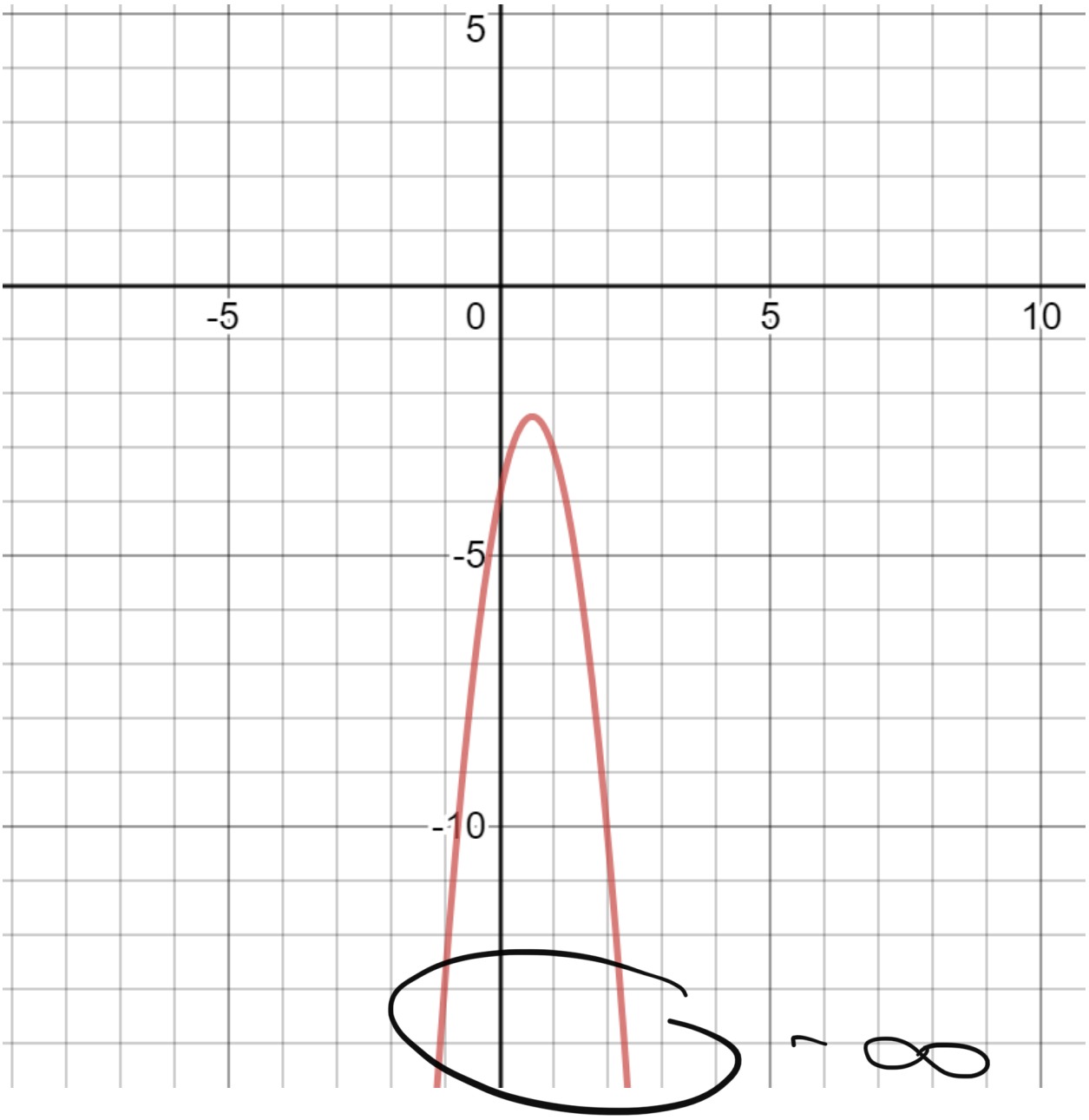
35) $\lim_{x\to\infty}(-2x+6)$ $-\infty$



37) $\lim_{x\to\infty}(4x^2+5x-4)$ $\simeq \infty$



39) $\lim_{x\to\infty}(-4x^2+5x-4)$ $= -\infty$



$-\infty$